A comparative study of image compression based on directional wavelets

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ABSTRACT

Discrete wavelet transform is an effective tool to generate scalable stream, but it cannot efficiently represent edges which are not aligned in horizontal or vertical directions, while natural images often contain rich edges and textures of this kind. Hence, recently, intensive research has been focused particularly on the directional wavelets which can effectively represent directional attributes of images. Specifically, there are two categories of directional wavelets: redundant wavelets (RW) and adaptive directional wavelets (ADW). One representative redundant wavelet is the dual-tree discrete wavelet transform (DDWT), while adaptive directional wavelets can be further categorized into two types: with or without side information. In this paper, we briefly introduce directional wavelets and compare their directional bases and image compression performances.

Keywords: scalable image coding, discrete wavelet transform, directional wavelets, redundant wavelets, dual-tree discrete wavelet transform, adaptive directional wavelets, side information, basis functions, compression performances, context-based arithmetic coding

1. INTRODUCTION

Because of the inherent heterogeneity of Internet, different networks have different channel characteristics, such as channel bandwidth, delay and jitter. In addition, terminals used by people are increasingly diversified with different display and processing abilities. Accordingly, users may receive varied image qualities through different networks and different terminals. Luckily, scalable image coding is able to solve the problem. Discrete wavelet transform (DWT) is an effective tool to generate scalable stream for images and JPEG2000 is just an excellent example of scalable coder based on discrete wavelet transform. It is capable of achieving high coding efficiency while providing scalable streams. However, conventional 2D DWT is a separable transform by performing 1D DWT along the vertical direction and then along horizontal direction, or horizontally and then vertically. Among the three resulting subband basis functions for each level, two have directional features (horizontal and vertical, respectively) but the third one mixes 45 degree with -45 degree. Hence, DWT cannot efficiently represent edges not aligned in horizontal or vertical directions. But natural images often contain rich edges and textures of this kind. Therefore, recently, intensive research has been focused particularly on the directional wavelets which can effectively represent directional attributes of images. Specifically, there are two categories of directional wavelets: redundant wavelets (RW) and adaptive directional wavelets (ADW).

One typical redundant wavelet is the dual-tree discrete wavelet transform (DDWT)[1] which has advantages of shift invariance and directional selectivity at a price of redundancy while we can get compact expression using Noise Shaping Iterative Algorithm based on projection [2]. Adaptive directional wavelets can be categorized into two types: with or without side information. What’s more, adaptive directional wavelets with side information mainly have two sampling modes: orthogonal sampling[3] and quincunx sampling[4].

This paper is organized as follows. Section 2 briefly introduces directional wavelets, including redundant wavelets and adaptive directional wavelets. Section 3 compares the directional bases and image compression performances of directional wavelets. Finally in Section 4 we summarize our work and discuss the future work for image coding using DDWT.
2. DIRECTIONAL WAVELETS

2.1 Redundant wavelets

One representative redundant wavelet is the dual-tree discrete wavelet transform (DDWT). DDWT is a complex wavelet transform. Taking HH subband for example, the corresponding wavelet function is

$$\psi(x, y) = \psi(x)\psi(y),$$  \hspace{1cm} (1)

where,

$$\psi(x) = \psi_h(x) + j\psi_g(x)$$  \hspace{1cm} (2)
$$\psi(y) = \psi_h(y) + j\psi_g(y)$$  \hspace{1cm} (3)

and $\psi_g(t)$ is the Hilbert transform of $\psi_h(t)$. In order to reduce the redundancy degree, we get the real part of $\psi(x, y)$ only.

$$\text{Real}[\psi(x, y)] = \psi_h(x)\psi_h(y) - \psi_g(x)\psi_g(y)$$  \hspace{1cm} (4)

Equation (4) can be regarded as the subtraction of HH subband wavelets of two 2D DWT. Other subband wavelets can be obtained in the same way. Hence, DDWT’s directional wavelets are actually the result of addition/subtraction of two separable 2D DWT subbands, whose scaling function and wavelet function meet Hilbert transform respectively. Pyramid decomposition shown in Fig. 1 is used on these two separable wavelets respectively, which forms the dual-tree structure as represented in Fig. 2.
2D or higher dimension DDWT has advantages of shift invariance and directional selectivity. For example, 2D DDWT has six directionally selective subbands which orient on 15°, 45°, 75°, 105°, 135°, 165° respectively. However, the benefits of DDWT come at a price of redundancy. The redundancy degree of d dimension DDWT is $2^d : 1$, and can be reduced to $2^{d-1} : 1$ by retaining the real part only. Though the coefficients of 2D DDWT is redundant, we can get more compact expression using Noise Shaping Iterative Algorithm based on projection [2].

### 2.2 Adaptive directional wavelets

Adaptive directional wavelet transform incorporates the directionally spatial prediction or update into the conventional lifting-based wavelet transform. It can be categorized into two types: with or without side information.

Adaptive directional wavelets with side information mainly have two sampling modes: orthogonal sampling [3] and quincunx sampling [4], as shown in Fig. 3. Quincunx sampling firstly splits samples of the image into two parts—$X_e$ (represented by "○") and $X_o$ (represented by the "●") as shown in Fig. 3(a), and do the prediction step followed by the update step. In the prediction step, $X_o$ is predicted by the linear combination of $X_e$ with 8 selective directions. The direction with minimal prediction error is chosen. In the update step, $X_e$ is updated by the linear combination of $X_o$ with the same direction which is employed in the prediction step. Orthogonal sampling is similar to quincunx sampling except for the splitting step, as shown in Fig. 3(b). Orthogonal sampling is asymmetrical; It needs to do adaptive lifting twice, row first or column first, the order of which can affect the ability of de-correlation. On the other hand, quincunx sampling is symmetrical, and does not have such a problem. The cost of adaptive directional wavelets with side information is to transmit the direction information. In order to reduce the side information, we can choose the same direction for each block rather than each pixel. Each block includes $2^nx2^n$ samples, and the direction is chosen to minimize the sum of absolute prediction errors of all the samples in the block.

![Fig. 3. Adaptive directional wavelets with side information](image)

(a) Quincunx sampling  (b) Orthogonal sampling

Adaptive directional wavelets without side information [5] shown in Fig. 4 also do adaptive filtering based on the lifting scheme (row first or column first) on the orthogonal sampling grid. The selective directions is $0°, 45°, 135°$, and it evaluates the difference between the two predicting pixels in each selective direction (for example, $\Delta_{45} = |x[m+1,2n-1]-x[m-1,2n+1]|$), and chooses the direction with the minimal difference. The decoder selects the direction using the same algorithm so that there is no mismatch. However, this method is very sensitive to quantization, thus having lower reconstruction quality.
3. EXPERIMENTAL RESULTS

In this section, we compare the aforementioned directional wavelets in terms of directional basis functions and image compression performances.

3.1 Comparison of basis functions

We compare the basis functions of DWT, DDWT and adaptive directional wavelets. As shown in Fig. 5(a), Conventional DWT has one horizontal subband, one vertical subband, and a mixed-direction subband with checkerboard pattern. Therefore, DWT cannot efficiently represent edges not aligned in horizontal or vertical directions. DDWT has six directionally selective subband basis functions—15°, 45°, 75°, 105°, 135°, 165°, given in Fig. 5(b). The basis functions of adaptive directional wavelets shown in Fig. 5(c)(d) are consistent with their selective directions (Note that the basis functions shown for orthogonal sampling are after the column wise decomposition only). It can be seen that directional wavelets have more direction selectivity.

Fig. 5. Comparison of directional basis functions
(a) DWT                           (b) DDWT  
(c) Orthogonal sampling  (d) Quincunx sampling
3.2 Comparison of compression performances

We incorporate DDWT and adaptive directional wavelets with/without side information into the JPEG2000 reference software “JasPer”\[6\]. Here we compare their coding efficiency with original JPEG2000 which uses conventional DWT. Four test images, Barbara, Lena, Boat and Peppers with a size of 512x512 are used in this experiment.

The first experiment evaluates the trade-off between the rates used for specifying the direction information and the reconstruction quality with adaptive directional wavelets with side information. In order to reduce the direction information, we use blocks instead of samples. Each block includes 2^n x 2^n samples, and the prediction direction for each block is chosen to minimize the sum of absolute prediction errors of all the samples in the block. We code the direction information using de-correlation and arithmetic coding. Fig. 6 is the rate-distortion curve under 6 different block sizes without considering bits for coding the directional information. It can be seen that going from pixel-wise selection to even the smallest block size of 2x2 induces a significant loss in prediction accuracy, but the performance discrepancies between using 2x2, 4x4, 8x8, 16x16 and 32x32 blocks are small. Fig. 7 shows the rate-distortion curve considering directional information cost. It can be seen that the best block size is 16x16 for orthogonal sampling and is 8x8 for quincunx sampling. But the performance difference among using block sizes from 8x8 to 32x32 is relatively small.

In the second experiment, we compare the compression performances obtained using different transforms. Image decomposition level is 5 for all the transforms. Orthogonal sampling uses 16x16 mode, and quincunx sampling uses 8x8 mode. For DDWT, we take only the real part of complex coefficients, reducing the redundancy degree from 4:1 to 2:1. Coefficients are deduced with Noise Shaping Iterative Algorithm based on projection \[2\]. The initial threshold is 128, which is reduced to 1 with a stepsize of 1 iteratively. And the gain factor \( \hat{k} \) is set to 1.6 as suggested in \[2\]. The DDWT has 6 highpass subbands and 2 lowpass subbands for each level of decomposition, and we treat every two adjacent subbands of DDWT as a single subband in the JPEG2000 codec for implementation convenience. Rate-distortion curves for four test images are given in Fig. 8. Adaptive directional wavelets using orthogonal sampling with side information outperforms conventional DWT of JPEG2000 by 1.25dB in Barbara, 0.3dB in Lena, 0.25dB in Boat and 0.15dB in Peppers, and outperforms DDWT 0.8dB in Barbara and Lena, 1.1dB in Boat and 0.9dB in Peppers. On the whole, directional wavelets have good performances and the performances of the adaptive wavelets using quincunx sampling and DDWT are slightly worse than that with orthogonal sampling. The reason for this is still under investigation. Fig. 9 illustrates the reconstructed images of DWT, DDWT and adaptive directional wavelets at the bit rate of 0.2 bpp. It can be seen that the image reconstructed by adaptive directional wavelets without side information has the largest distortion, primarily because of its sensitivity to quantization. After the quantization, it is difficult for decoder to correctly find the same direction as encoder. Due to the redundancy of DDWT and the lack of specifically optimized subband coding, the quality of reconstruction is not good enough at the same bit rate. In the reconstructed image of conventional DWT, edges are not smooth. Adaptive directional wavelets with side information have the best reconstruction quality with smooth edges.

![Fig. 6. Rate-distortion curve under different block sizes with directional wavelet transform](without considering directional information cost)  
(a) Orthogonal sampling    (b) Quincunx sampling
Fig. 7. Rate-distortion curve under different block sizes with directional wavelet transform (with considering directional information cost)
(a) Orthogonal sampling  (b) Quincunx sampling

Fig. 8. Comparison of Codecs Using Different Wavelet Transforms
(a) Barbara    (b) Lena    (c) Boat    (d) Peppers
Fig. 9. The reconstructed images at the bit rate of 0.2 bpp
(a) Orthogonal sampling  (b) Quincunx sampling  (c) DDWT
(d) ADW without side information  (e) DWT
4. CONCLUSION

In this paper, we briefly introduce directional wavelets and compare their directional basis functions and image compression performances. Our experiments show that adaptive directional wavelets without side information has the worst performance. This is because its directional selectivity is significantly worse than with side information, the selectivity is further affected by quantization. On the whole, directional wavelets have good performances. DDWT can achieve good directional selectivity, but the cost is redundancy. Though the Noise Shaping Algorithm helps to reduce the number of coefficients to be coded, more bits are needed to code the location of the significant coefficients. The current coder for DDWT does not exploit the correlation between redundant subbands. The study in [7] shows that there is strong correlation in the location information of significant coefficients and the codec in [8] used vector coding across subbands to code the location information. We expect that exploiting cross-subband correlation using a method similar to [8] would improve the coding efficiency of the DDWT codec. Also, treating two subbands of DDWT with two different directions as a single subband in our current implementation does not allow the codec to exploit the redundancy along the two different directions efficiently. Finally the context used in the JPEG2000 codec was designed based on the direction pattern of standard DWT, not that of DDWT. Modification of context-based arithmetic coding based on the direction features of the DDWT may also improve the coding efficiency.

In our future work, we will research on the modification of context-based arithmetic coding based on the direction features of directional wavelets which may improve their compression performances.

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